REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (leave blank)

2. REPORT DATE
Sept. 4, 1996

3. REPORT TYPE AND DATES COVERED
FINAL Report, July-September, 1996

4. TITLE AND SUBTITLE
A Conference on "Wavelets, Relations with Operators and Applications".

5. FUNDING NUMBERS
F49620-96-1-0365

6. AUTHOR(S)
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AFRL-SR-BL-TR-98-
0128

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
UNC-Charlotte
Charlotte, NC 28223

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
110 Duncan Avenue, Suite B115
Bolling AFB, DC 20332

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Distribution Unlimited

12b. DISTRIBUTION CODE

D

13. ABSTRACT (Maximum 200 words)

During July 24-28 there was a conference on "Wavelets, relations with operators and applications". Funding for this conference was provided by AFOSR, NSF and UNCC.

There were 64 (including 3 from UNCC) participants registered for the Conference. This was an international conference. Participants are from eleven countries (including US). We exchanged ideas with the Guido Weiss' group and David Larson's group and many other participants. New ideas and new problems arose.

The conference proves that abstract mathematics (including operator theory, operator algebras, number theory, group theory, Banach space theory) provided and will provide essential contribution into wavelet theory. The talks covered a vast area of mathematics including: wavelets, frames, Banach space theory, numerical analysis, signal processing, image analysis, quantum optics, operator theory, operator algebras, harmonic analysis, lie group, number theory, dynamical system, partial differential equations and approximation theory.

19980205 045

14. SUBJECT TERMS
Waveltes, Operator Theory, Operator Algebras

DTIC QUALITY INSPECTED

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

Scientific Report on Gransts: F49620-96-1-0103

The Conference on " Wavelets, relations with operators and applications was held July 24-28, 1996 in UNC-Charlotte. The event was organized and hosted by the University of North Carolina at Charlotte. The organizer is Xingde Dai. Funding for the conference was provided by AFOSR ($ 5,300) , NSF ($ 5,300) and UNCC ($ 2,000). There were 64 (including 3 from UNCC) participants registered for the Conference. This was an international conference. Participants are from eleven countries (including US). The participants who came from countries other than US were on their own travel expenses. (We provided housing and part of food money for two participants from Russia and China with registration fees wavered.)

This was a great conference. The idea "mathematics behind the wavelet theory attracts some very big names. We have three two hour speakers, nine one hour speakers and twenty six half hour speakers. The main speakers and their titles of speeches are:

Two hour speakers

1. **Guido Weiss (Washington University)**
   - Unified approach to wavelets, the ϕ and ψ transform and other related systems (I);
   - Unified approach to wavelets, the ϕ and ψ transform and other related systems (II).
2. **David Larson (Texas A&M University)**
   - Wandering vectors for unitary systems and orthogonal wavelets
   - Operator-theoretic interpolation of wavelets
3. **Charles Chui (Texas A&M University)**
   - A discussion of local bases
   - Wavelets for Signal and image analyses

One hour speakers

1. **John J. Benedetto (University of Maryland, College Park)**
   Noise reduction and sampling multipliers in frame decompositions
2. **Brian DeFacio (University of Missouri-Columbia)**
   Sampling,aliasing and wavelet families for quantum optics

3. **Eugenio Hernandez (Universidad Autonoma de Madrid, Spain)**
   Smoothing Minimally Supported Frequency (MSF) wavelets and invariant cycles.

4. **Lawrence Baggett (University of Colorado-Boulder)**
   Some Pure Math Spinoff from Wavelet Theory

5. **Steen Pedersen (Wright State University)**
   Harmonic analysis of iterated function systems

6. **Peng Lizhong (Beijing University, China)**
   Admissible wavelets on the Siegel domains and Toeplitz-Hankel Operators

7. **Joachim Stoeckler (Universitaet Duisburg, Germany)**
   A Laurent operator technique for affine frames.

8. **Jiangzhong Wang (Sam Houston State University)**
   Operators of Toeplitz Type and Their Application in Wavelet Analysis

9. **Yang Wang (Georgia Institute of Technology)**
   Self-Affine Tiles and Haar Wavelets in $\mathbb{R}^n$

Henry Landau (AT&T) attended the conference. Joe Cima (UNC-Chapel Hill) registrated to the conference but was not be able to make it. More then one hundred people showed interest in attending the conference. Our program was so compact so we even could not let Wayne Lawton to give a one hour talk.

The Conferencep proves that abstract mathematics (including operator theory, operator algebras, number theory, group theory, Banach space theory) provided and will provide essential contribution into wavelet theory. The talks convered a vast area of mathematics encluding: wavelets, frames, Banach space theory, numerical analysis,signal processing, image analysis, quantum optics, operator theory, operator algebras, harmonic analysis, lie group, number theory, dynamical system, partial differential equations and approximation theory.

Some young graduate students (we have 10) presented their high quality research works.

Some new research partnerships were initiated. The group of Guido Weiss, the group of David Larson (Texas A&M University) and the group of UNC-Charlotte had a joint two hour meeting in the evening of Friday, July 26. (In total 14 people). During the meeting they discussed mathematics and colaboration in nearly future. David Larson has a new joint project with Steen Petersen. Xingde Dai answered a question by Guido Weiss. Many people found right people to talk with during the conference.

It short, I believe the gaol of the conference has beed reached.

Xingde Dai

Principal Investigator

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Conference Schedule

All talks will be held in auditorium in the Storrs Architecture Building, which is very close to the parking deck C (on Mary Alexander Dr.). Each morning, July 25-28, at 8:30am, there will be a continental breakfast at Storrs Achitecture building. Beverages and refreshments will be provided later each day.

PLEASE DO NOT TAKE FOOD OR DRINKS INTO THE AUDITORIUM!

July 24/Wednesday Afternoon

1:00PM Opening. Schley R. Lyons (Dean of College of Arts & Sciences, UNCC)

Welcome

1:30-2:20 Guido Weiss (Washington University)

Unified approach to wavelets, the ϕ and ψ transform and other related systems (I)

2:30-3:20 John J. Benedetto (University of Maryland, College Park)

Noise reduction and sampling multipliers in frame decompositions

3:30-4:00 Break

4:00-4:20 Peter G. Casazza (University of Missouri-Columbia)

Finite Dimensional Methods For Frame Theory

4:30-4:50 Manos Papadakis (Hellenic Military Academy, Greece)

Unitary mappings between MRA and a parametrization of low-pass filters

5:00-5:20 Raj Sharman (and John Tyler) (LSU)

Registration of Images using Wavelets

5:30-5:50 John Weiss (Prometheus, Inc.)

Thermal Analysis of Multichip Modules using the Wavelet-Galerkin Method

6:00 Dinner time

July 25/Thursday Morning

9:00-9:50 Brian DeFacio (University of Missouri-Columbia)

Sampling,aliasing and wavelet families for quantum optics

10:00-10:50 David Larson (Texas A&M University)

Wandering vectors for unitary systems and orthogonal wavelets

11:00-11:20 Rufeng Liang (UNC-Charlotte)

Some results on wavelets: their multipliers, phases and paths

11:30-11:50 Ole Christensen (Technical University of Denmark, Denmark)

Applications of pseudo-inverse operators in frame theory

12:00 Lunch

July 25/Thusday Afternoon

1:30-2:20 Guido Weiss (Washington University)

Unified approach to wavelets, the ϕ and ψ transform and other related systems (II)

2:30-2:50 Yongsheng Han (Auburn University)

Calderon reproducing formula and sampling theorem

3:00-3:20 Carl Taswell (Stanford University)

Wavelet Transform Convolution Versions for Fast Wavelet Based Numerical Algorithms

3:30-4:00 Break

4:00-4:20 Jeff Hogan (Macquarie University, Australia)

Wavelet Bi-frames with Unilateral Vanishing Moments

4:30-4:50 Darrin Speegle (Texas A&M University)

A Construction of Wavelet Sets in $\mathbf{R}^n$

5:00-5:20 Vishnu Kamat (Texas A&M University)

Operators and Multi-wavelets

5:30-5:50 Wayne Lawton (National University of Singapore, Singapore)

Refinable Distributions on Lie Groups

6:00 Dinner time

July 26/Friday Morning

9:00-9:50 Yang Wang (Georgia Institute of Technology)

Self-Affine Tiles and Haar Wavelets in $\mathbf{R}^n$

10:00-10:50 Charles Chui (Texas A&M University)

A discussion of local bases

11:00-11:20 Joe Ward (Texas A&M University)

Nonstationary Wavelets on the m-Sphere for Scattered Data

11:30-11:50 Eugen Ionascu (Texas A&M University)

On the unitary systems affiliated with multi-variated wavelet theory.

12:00 Lunch

July 26/Friday Afternoon

1:30-2:20 Lawrence Baggett (University of Colorado-Boulder)

Some Pure Math Spinoff from Wavelet Theory

2:30-3:20 Jiangzhong Wang (Sam Houston State University)

Operators of Toeplitz Type and Their Application in Wavelet Analysis

3:30-4:00 Break

4:00-4:20 Shinzo Kawamura (Yamagata University, Japan)

Chaotic dynamical system and bases of Walsh type in Hilbert spaces

4:30-4:50 Michael Zeitlin (Russian Academy of Sciences, Russia)

Wavelets in dynamics, optimal control and routes to chaos in Galerkin approximations

5:00-5:20 Oleg V. Vasilyev (Minnesota Supercomputer Institute)

Adaptive multilevel wavelet collocation method for solving partial differential equations in multiple dimensions

5:30-5:50 Tim Flaherty (University of Pittsburgh)

Multi-wavelets in Two Dimensions

6:00 Dinner time

July 27/Saturday Morning

9:00-9:50 Eugenio Hernandez (Universidad Autonoma de Madrid, Spain)

Smoothing Minimally Supported Frequency (MSF) wavelets and invariant cycles.

10:00-10:50 David Larson (Texas A&M University)

Operator-theoretic interpolation of wavelets

11:00-11:20 Qing Gu (Texas A&M University)

Existance of certain types of interpolation families of wavelet sets.

11:30-11:50 Wei Cai (UNC-Charlotte)

Simulations of laminar flames with wavelet collocation methods.

12:00 Lunch

July 27/Saturday Afternoon

1:30-2:20 Steen Pedersen (Wright State University)

Harmonic analysis of iterated function systems

2:30-3:20 Charles Chui (Texas A&M University)

Wavelets for Signal and image analyses

3:30-4:00 Break

4:00-4:20 Gustavo Garrigos (Washington University)

Some connectivity results in wavelets.

4:30-4:50 Ming-Jun Lai (University of Georgia-Athens)

Examples of Bivariate Nonseparable Compactly Supported Orthonormal Wavelets

5:00-5:20 Deguang Han (Texas A&M University)

Irrational rotation unitary operator systems

5:30-5:50 Jeff Knisley (East Tennessee State University)

A Method for Estimating the Amplitudes and Frequencies of Gabor Wavelet Decompositions

6:00-6:20 Zhiping He (Canada)

Wavelet multipliers and Signals

6:40 Shuttle (van, at deck D) (for schedule see the sheet "Shuttle Schedule") to Hot Wok Restaurant, Town Center Plaza. Following diner, shuttle service back to hotels and Martin Village apts will begin at 9:00pm.

July 28/Sunday Morning

9:00-9:50 Peng Lizhong (Beijing University, China)

Admissible wavelets on the Siegel domains and Toeplitz-Hankel Operators

10:00-10:50 Joachim Stoeckler (Universitaet Duisburg, Germany)

A Laurent operator technique for affine frames.

11:00-11:20 Tian-Xiao He (Illinois Wesleyan University)

Spline wavelet transform

End of Conference

Abstracts of Talks at Conference

–Wavelets, Relations with Operators and Applications–

Sponsered by AFOSR, NSF abd UNCC

at UNC-Charlotte, Charlotte, North Carolina, USA
July 24-28, 1996

Programmer: Xingde Dai (UNC-Charlotte)

1. **Two hour speakers**

   (a) **Guido Weiss (Washington University)**

   - Unified approach to wavelets, the ϕ and ψ transform and other related systems (I);
   - Unified approach to wavelets, the ϕ and ψ transform and other related systems (II).

   Two equations involving the Fourier transform of an $L^2(\mathbf{R}^n)$ function (or a finite collection of such functions) completely characterize all the wavelets and related functions in n-dimensional Euclidean space. In particular, this applies to the phi-transform expansions of Frazier and Jawerth. This characterization extends to the case when the analyzing and synthesizing functions do not coincide. Applications of this characterization will also be given.

   (b) **David Larson (Texas A&M University)**

   - Wandering vectors for unitary systems and orthogonal wavelets
   - Operator-theoretic interpolation of wavelets

   A wavelet set is a measurable subset of the Real line whose normalized characteristic function is the Fourier-Plancherel transform of a dyadic orthonormal wavelet. The corresponding wavelets are analogues of Shannon's wavlet. There are sufficiently many wavelet sets to generate the Borel structure of R. An arbitrary pair (E,F) of wavelet sets determines a measure-preserving transformation of R whose affiliated composition operator plays a central role in the operator-theoretic approach to wavelet theory that the speaker has developed over the past few years jointly with X. D ai of UNC-Charlotte. When this transformation is involutive (E,F) is called an interpolation pair. In this case using operator techniques we obtain a 2X2 function-matricial criterion which completely characterizes the set of all dyad ic orthonormal

wavelets whose Fourier transforms are supported in the union of E and F. This answers affirmatively a problem of Dai-Larson, and also generalizes a recent result of E. Hernandez, X. Wang and Guido Weiss on smoothing of wavelets.

(c) **Charles Chui (Texas A&M University)**

- A discussion of local bases
- Wavelets for Signal and image analyses

1. This is a joint work with X. L. Shi. Motivated by the need of effective time-frequency localization techniques, "wavelets" has become a rapidly developing mathematical subject in the field of computational harmonic analysis. While the short-time Fourier transform has been a very popular tool for time-frequency localization, the rigidity of its time-frequency window gives rise to some limitation to its applications to many engineering applications. On the other hand, although the wavelet transform has the zoom-in and zoom-out capability, it is still a constant-Q filter. Adaptive time-frequency analysis requires more sophisticated mathematical tools. The localized cosine transform seems to be an attractive adaptive technique. In this direction, we give complete characterizations of localized trigonometric systems to be Schauder bases, Riesz bases, and frames, and we also give explicit formulations of their duals.

2. Wavelets will be introduced from the point of view of time-frequency and time-scale localization. The search of near- optimal wavelets for bandpass filtering is critical for high-quality performance in medical image analysis. We will disuss applications of the wavelet transform to medical image compression and computer-aided diagnosis.

2. One hour speakers

(a) **John J. Benedetto (University of Maryland, College Park)**
Noise reduction and sampling multipliers in frame decompositions

A characterization of frames of translates is used to construct new wavelet frames. The associated multirate systems provide best possible coding/quantization noise reduction. The inherent oversampling in wavelet frames motivates new proofs and formulations of the Poisson Summation Formula for classical function spaces. These lead to the use of multiplier theory to obtain new frame decompositions in terms of generalized regular sampling. The first part is joint work with Shidong Li, and the second part is joint work with Georg Zimmermann.

(b) **Brian DeFacio (University of Missouri-Columbia)**
Sampling,aliasing and wavelet families for quantum optics

Sampling,aliasing and wavelet families for quantum optics A few observations on relations between wavelets and aliasing for compactly supported ,orthonormal families are given.These include Haar,Shannon,Daubechies.least asymmetric Daubechies,Coiflets,Franklin splines and others.The coherent and squeezed states of quantum optics are introduced in terms of Fock space quantities. The fact that coherent states generate complex valued translations and that squeeze operators generate uncertainty preserving scale changes of a signal will be reviewed.The aliasing structure will then be shown to provide 'natural wavelet families'with particularly good sampling features. Physical interpretations and possible applications will be mentioned.

(c) **Eugenio Hernandez (Universidad Autonoma de Madrid, Spain)**
Smoothing Minimally Supported Frequency (MSF) wavelets and invariant cycles.

The Fourier transforms of the Lemarié-Meyer wavelets can be viewed as a smooth aproximation to the Fourier transform of the Shannon wavelet. This result is not true in general for Minimally Supported Frequency (MSF) wavelets. Nevertheless, this can be done for a large number of MSF wavelets that arise from Multiresolution Analyses (MRA).

We shall describe a procedure to "mollify" the low-pass filters of a large number of MSF wavelets, so that the smoother functions obtained in this way are also low-pass filters for an MRA. Hence, we are able to aproximate (in the L^2-norm) MSF wavelets by wavelets with any desired degree of smoothness on the Fourier transform side. The MSF wavelets we consider are band-limited, but this may not be true for their smooth aproximations. This phenomena is related to the behaviour of the invariant cycles under the transformation $x \to 2x \pmod{2\pi}$. We shall present new examples of wavelets.

(d) **Lawrence Baggett (University of Colorado-Boulder)**
Some Pure Math Spinoff from Wavelet Theory

We demonstrate first that there is a connection between ergodic theory and number theory on the one hand and questions arising from windowed Fourier transforms on the other. Second, we formulate several possible abstract frameworks for a theory of orthonormal wavelets. We then apply these formulations to some examples including the hyperbolic plane.

(e) **Steen Pedersen (Wright State University)**
Harmonic analysis of iterated function systems

This talk summarizes work mostly joint with Palle E.T. Jorgensen. Starting with Fourier series in d-dimensional Euclidean space, orthogonal bases of exponentials for various domains are constructed using a pair of affine functions, one for the domain and a dual affine function for the set of frequencies. The construction is iterated, at each step we have a domain and an orthogonal basis of exponentials for the corresponding L^2 space, in the limit the domain is replaced by a fractal measure and the basis of exponential by a fractal in the large (and also by a dual fractal). Interactions between the two limit objects are investigated. The same iteration process is applied to certain frames of exponentials yielding the same limit objects, but different results about the limit objects.

(f) **Peng Lizhong (Beijing University, China)**
Admissible wavelets on the Siegel domains and Toeplitz-Hankel Operators

Admissible wavelet comes from square-integrable representation of nonunimodular group. Let G be a semi-simple Lie group (which is unimodular) and G=ANK be its Iwasawa decomposition, where K is the maximal compact subgroup. G/K can be realized as a Cartan or Siegel domain D. The affine group P=AN is nonunimodular, it has a representation on the square-integral function space on the boundary of D. The square- integrable elements of this representation give the admissible wavelets. The orthogonal basis of admissible wavelets is constructed, and the complete orthogonal decomposition of the square-integrable function space on D is given. The first component turns to be the Bergman space. Two important examples, real line and the Heisenberg group, are provided. By the orthogonal projections into the subspaces, three series of Toeplitz- Hankel type operators are defined, it is showed that they are just paracommutators, and their boundedness, compactness, Schatten-von Neumann properties and cut-off properties are given.

(g) **Joachim Stoeckler (Universitaet Duisburg, Germany)**
A Laurent operator technique for affine frames.

We study affine frames of $L_2(\mathbf{R}^s)$ which are generated by a finite family of functions $\Psi = \{\psi_1, \ldots, \psi_L\}$ and a scaling matrix $M \in \mathbf{Z}^{s\times s}$. The 'analysis' operator T_Ψ maps $f \in L_2(\mathbf{R}^s)$ to the sequence of inner products with the frame elements. Its adjoint T_Ψ^* is the so-

called 'synthesis' operator and maps sequences in ℓ_2 to corresponding combinations of the frame elements.

The basic idea of our development is the following: Given two affine frames (generated by families Ψ and Θ with the same cardinality L), the operator product

$$S := T_\Psi \circ T_\Theta^*$$

commutes with the bilateral shift of infinite multiplicity; more precisely, S is a generalized Laurent operator which commutes with the shift on the Hilbert space $\ell_2(H)$ where $H = \ell_2(\Psi \times \mathbf{Z}^s)$. This fact is a direct consequence of the invariance of affine frames with respect to dilation by powers of the scaling matrix M.

We give three different applications of this 'Laurent-operator technique'. Firstly, the recent characterization of minimally supported orthonormal wavelets by Hernandez et.al. is extended to Riesz bases. As a side product we prove that no inexact frames with this restricted support of the Fourier transform exist. Secondly, a characterization of univariate tight affine frames with dyadic scaling is obtained from our representation. This gives a new proof for a special case of a result by Ron and Shen. As our third application we present a fast pyramidal algorithm for frames which are obtained from oversampling of biorthogonal wavelet bases. Here, the generalized Laurent operators S and S^*S have simple explicit representations. This is very useful for the preconditioning of the frame operator whose inverse is needed for frame decompositions of functions in $L_2(\mathbf{R}^s)$.

REFERENCES

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Hernandez,E., X.Wang and G.Weiss, Smoothing minimally supported frequency (msf) wavelets, parts 1 and 2, preprint, 1995.

Ron, A., and Z. Shen, Affine systems in $L_2(\mathbf{R}^d)$: the analysis of the analysis operator, preprint, CMS Report 96-02, University of Wisconsin, Madison, 1996.

Stöckler,J., Multivariate affine Frames, Habilitationsschrift, University of Duisburg, 1995.

(h) **Jiangzhong Wang (Sam Houston State University)**

Operators of Toeplitz Type and Their Application in Wavelet Analysis

Let G be an Abelian group of linear operators on a linear space S. S is said to be an invariant space under G, if for any point $g \in G$,

$g(S) \subset S$. S is said to be finitely generated if there are finite set $F \subset S$ such that $S = \mathrm{span}\,(G(F))$. A linear operator t on S is said to be a Toeplitz type operator if, for any $g \in G, gt = tg$. In wavelet theory, a lot of operators, such as transition operator, subdivision operator, wavelet transform, etc., can be considered as Toeplitz type operators. In this paper, we study the properties of this kind of operators and reveal that many properties of scaling functions, such as stability and linear independence, approximation order, are relevant to certain properties of such Toeplitz type operators that is commutative with the shift operator in sequence spaces.

(i) **Yang Wang (Georgia Institute of Technology)**
Self-Affine Tiles and Haar Wavelets in $\mathbf{R}^n$

A self-affine tile is a tile which has the property that certain affine image of the tile can be tiled perfectly by using the original tile. In this talk I shall give an overview and present the latest results in this area. I shall also discuss how self-affine tiles are related to multiresolution analysis and orthogonal wavelets. Several open problems in the area shall be presented.

3. **Half hour speakers**

(a) **Peter G. Casazza (University of Missouri-Columbia)**
Finite Dimensional Methods For Frame Theory

Let $(f_i)_{i=1}^{\infty}$ be a frame for a Hilbert space H and P_n the orthogonal projection onto $(f_i)_{i=1}^{n}$. The frame is said to satisfy the **strong projection method** if $\forall f \in H$, the frame coefficients for $P_n f$ in $(f_i)_{i=1}^{n}$ converge in ℓ_2-norm to the frame coefficients for f. These are precisely those frames for which finite dimensional methods (applied to finite subsets of the frame) yield the global properties of the frame. We give complete (and easily applied) characterizations of when the strong projection method works for frames. As a result, we answer a long list of open questions in this area. We also give examples to show that all the results are best possible. This is joint work with Ole Christensen.

(b) **Yongsheng Han (Auburn University)**
Calderon reproducing formula and sampling theorem

Using Calderon-Zygmund operator theory, we obtain more general discrete C aldeon type reproducing formula. This formula is similar to the Shannon sampling theorem. However, the proof of this formula doesn't use the Fourier transform, either translation and dilation. So

this formula still holds on spaces of homoge neous type introduced by Coifman and Weiss. As an application, we establish the Plancherel-Polya type inequality".

(c) **John Weiss (Prometheus, Inc.)**

Thermal Analysis of Multichip Modules using the Wavelet-Galerkin Method

The reliability assessment of multichip modulesis is conditioned by their high cost, low production levels, and high complexity. Together these conditions make traditional statistical based methods of testing inoperative. The solution is to develop the reliability assesssment process during the design process through intelligent enginneering procedures. An important component of this effort is the finite element thermal and stress analysis of the multichip design. In fact this component is the critical component in terms of speed and accuracy. Since flaws in a design tend to occur at material interfaces the accurate resolution of the thermal diffusion and induced stress effects at these singular interfaces is of great importance. The accuracy assessment has been poor due to singularity issues. Development of computational algorithms and software the accurate calculation of strains and stress near singularaties remains an undeveloped, but vitally needed, component of complete MCM reliability assessment capability. Accurate and practical numerical calculations would be of great use in the implementation and extension of modules for the reliable design of multichip modules.

We have found that the wavelet-Galerkin method can numerically resolve singularities while maintaining accuracy for smooth components of a solution. For instance, the wavelet-Galerkin method has numerically resolved shocks, turbulence, singular particle potentials, and the effects of boundary geometry with excellent results. We expect the wavelet-Galerkin method can produce a considerable improvement in the speed and accuracy of the thermal analysis of multichip modules.

We have developed a wavelet-Galerkin algorithm for the solution of the initial value problem for the Heat diffuson equation in a finite, composite multidimensional region. The heat flow and the possibly singular coefficients of diffusion are resolved by wavelet-Galerkin expansions. The results of numerical calculations are presented. Even with widely varying time and space scales, the numerical algorithm converges with regard to both wavelet order and discretization. For a piecewise constant coefficients of diffusion the wavelet-Galerkin algorithm has an exact reduction to an almost convolutional form. This leads to a fast (as FFT) algorithm.

(d) **Raj Sharman (and John Tyler) (LSU)**
Registration of Images using Wavelets

Registration of medical images is a very important issue in Medical Imaging. Other uses of registration can be found in GIS for registering satellite images. I am using wavelets to improve on the process of registration. My talk will explain what is registration, why it is important, review a few registration techniques and explain how we are using wavelets to do medical image registration. And how this registration gives better results in image compression. Our technique uses wavelets to do this.

(e) **Rufeng Liang (UNC-Charlotte)**
Some results on wavelets: their multipliers, phases and paths

We characterize the wavelet multipliers and have an algorithm to construct all of them; we characterize the phases of all MRA wavelets; we prove that there is a large path-connected subset of MRA wavelets which contains all known "good" wavelets.

(f) **Ole Christensen (Technical University of Denmark, Denmark)**
Applications of pseudo-inverse operators in frame theory

Applications of pseudo-inverse operators in frame theory: A classical result of Paley-Wiener states that if a Riesz basis is perturbed slightly, then we get a Riesz basis again. In the talk we show that a much more general result about frames can be proved using recent developements in the theory for pseudo-inverse operators.

(g) **Darrin Speegle (Texas A&M University)**
A Construction of Wavelet Sets in R^n
Given a strictly expansive matrix D and translations T_i by multiples of a constant in each coordinate direction, a *wavelet set* in R^n is a measurable set $E \subset R^n$ such that $\{D^n(E)\}_{n=-\infty}^{\infty}$ and $\{T_i(E)\}_{i=-\infty}^{\infty}$ are partitions of R^n. Indicator functions on such sets are Fourierof wavelets. We give a sufficient condition for a set A to be contained in some (not necessarily bounded) wavelet set E, and make several obesrvations based on the construction of E. Among these, we prove that there are always wavelet sets in R^n (hence, single- function wavelets) and that the collection of all wavelet sets is path- connected.

(h) **Wayne Lawton (National University of Singapore, Singapore)**

Refinable Distributions on Lie Groups

This paper constructs refinable distributions phi, on certain Lie groups G, as limits of cascade sequences $T^n f$. Here T is a refinement operator constructed from a pair (A,c) where A is a totally expansive automorphism of G, and c : Gamma -¿ C is a refinement mask defined on a discrete co-compact subgroup Gamma that is invariant under A. Necessary and sufficient conditions for $T^n f$ to converge in various topological vector spaces, and stability and regularity of ϕ, are related to eigenvalues and eigenvectors of operators derived from a transition operator W : C(Gamma) -¿ C(Gamma) constructed from (A,c).

(i) **Manos Papadakis ((Hellenic Military Academy, Greece)**
Unitary mappings between MRA and a parametrization of low-pass filters

We deal with the problem of determining all scaling functions in $L^2(\mathbf{R})$ associated with multiresolution analysis (MRA). We use unitary operators to map a given scaling function to any other scaling function of an MRA. We characterize these classes of unitary operators. We use this characterization to obtain all classes of Lebesgue measurable subsets K of the real line such that $(2\pi)^{-1/2}\chi_K$ is the Fourier transform of a scaling function. For obtaining classes of smoother in the frequency domain scaling functions. we are led to a parametrization of low-pass filters. We characterize the class of unitary operators acting on $L^2([-\pi,\pi)$ which map a given low-pass filter of an MRA to any other low-pass filter. Specifically we have: Set $Mf(t) = e^{it}f(t)$, $f \in L^2([-\pi,\pi))$. A low-pass filter associated with a scaling function is a function m in $L^\infty([-\pi,\pi))$ such that $\hat{\phi}(\gamma) = 2^{-1/2}m(\gamma/2)\phi(\gamma/2)$. If m_0 is the low-pass filter for a particular MRA then for every low-pass filter m corresponding to an MRA there exists a unitary operator W acting on $L^2([-\pi,\pi))$, commuting with M^2 such that $Wm_0 = m$. We expect that our techiques can be extended to the multidimensional case.

We recapture low-pass filters for Daubechies' scaling functions We also give an example of a scaling function which cannot be obtained by means of cascade algorithm. Finally we show by an example that the Dai-Larson parametrization of orthonormal wavelets does not apply to the generic problem of determining all scaling functions.

(j) **Eugen Ionascu (Texas A&M University)**
On the unitary systems affiliated with multi-variated wavelet theory.

The purpose of this paper is to show that in most cases, distinct dilation -translation wavelet systems on R^n give rise to unitarily inequivalent wavelet theories.

(k) **Shinzo Kawamura (Yamagata University, Japan)**
Chaotic dynamical system and bases of Walsh type in Hilbert spaces

We give a canonical covarinat representation of chaotic systems in one-dimensional dynamical systems such as quardratic maps on [0,1]. This representation shows that chaotic property of topological dynamical system is deeply relateted to the existence of a base of Walsh type and chaos implies the simple limit of the set of initial points (= a positive function with integral value 1), in contrast to that, they say, simple dynamical system implies complex future.

(l) **Michael Zeitlin (Russian Academy of Sciences, Russia)**
Wavelets in dynamics, optimal control and routes to chaos in Galerkin approximations

We give the explicit time description of the following problems: dynamics and optimal dynamics for nonlinear dynamical systems, Galerkin approximation for some class of partial differential equations and routes to chaos in Melnikov function approach to the perturbations of Hamiltonian systems. The first three problems and a part of fourth one are reduced to the problem of the solving of systems of differential equations with polynomial nonlinearities with or without some constraints. The first main part of our construction is some variational approach to this problem,which reduces initial problem to the problem of solution of functional equations at the first stage and some algebraical problems at the second stage. We consider also two private cases of our general construction. In the first case (particular) we have for Riccati equations the solution as a series on shifted Legendre polynomials, which is parameterized by the solution of reduced algebraical (also Riccati) system of equations. In the second case (general) we have the solution in a compactly supported wavelet basis. Multiresolution expansion is the second main part of our construction. The solution is parameterized by solutions of two reduced algebraical problems, one as in the first case and the second is some linear problem, which is obtained from one of the next wavelet construction: Fast Wavelet Transform (FWT), Stationary Subdivision Schemes (SSS), the method of Connection Coefficients (CC). Our general construction we use for solution important technical problems: minimization of energy in electromechanical system with enormous expense of energy and detecting signals from oscillations of a submarine (oscillations of a beam contacting with liquid). Next we

consider some details of the first particular case: synchronous drive of the mill–the electrical mashine with the mill as load. It is described by Park system of equations $\frac{di_k}{dt} = \sum_{r,s} A_{rs}i_r i_s + \sum_{\ell} A_\ell i_\ell + B_k$ where $A_{rs}, (r,s = \overline{1,6})$, $A_\ell, (\ell = \overline{1,6})$ are constants, $B_k(k = \overline{1,5})$ are explicit functions of time, $B_6(i_6, t) = a + di_6 + bi_6^2$ is analytical approximation for the mechanical moment of the mill. We use the next general form of energy functional in our electromechanical system $Q = \int_{t_0}^t [K_1(i_1, i_2) + K_2(\dot{i}_1, \dot{i}_2)]dt$, where K_1, K_2 are quadratic forms. We consider the optimization problem with some constraints, which are motivated by technical reasons. After the manipulations from the theory of optimal control, we reduce the problem of energy minimization to the some nonlinear system of equations. The obtained solutions are given in the next form : $i_k(t) = i_k(0) + \sum_{i=1}^{N} \lambda_k^i X_i(t)$, where in our first case we have $X_i(t) = Q_i(t)$, where $Q_i(t)$ are shifted Legendre polynomials and λ_k^i are roots of reduced algebraical system of equations. In wavelet case $X_i(t)$ correspond to multiresolution expansions in the base of compactly supported wavelets and λ_k^i are the roots of corresponding algebraical Riccati systems with coefficients,which are given by FWT,SSS or CC constructions. In CC method we compute them by combination of LU decomposition and QR algorithm. Also we use FWT and SSS for computing coefficients of reduced algebraical systems and for modeling D6–D10 functions and programs RADAU,DOPRI for testing. As a result we obtained the explicit time solution of optimal control problem. The generalization to polynomial systems is evidently. We use this method for the description of Galerkin approximations for beam equation and detecting chaotic and quasiperiodic regimes in naive Melnikov function approach to the perturbations of Hamiltonian systems.

Also, we consider symplectic Melnikov function approach. The following three concepts are equivalent: a square integrable representation U of a group G, coherent states over G, the wavelet transform associated to U. We have now three important cases:the affine group, which yields the usual wavelet analysis, the Weyl-Heisenberg group which leads to the Gabor functions, i.e. coherent states associated with windowed Fourier transform, also, now we have (Murenzi,Kalisa,Torresani) the case of bigger group, containing both affine and Weyl-Heisenberg group, which interpolate between affine wavelet analysis and windowed Fourier analysis: affine Weyl–Heisenberg group. But usual representation of it is not square–integrable and must be modified: restriction of the representation to a suitable quotient space of the group (the associated phase space in that case) restores square –

integrability: $G_{aWH} \longrightarrow$ homogeneous space.

Our goal is given another – symplectic – approach to this problem because of symplectic nature of our dynamical problem. Also, the symplectic structures (wavelets and Hamiltonian) must be consistent(this must be resemble the symplectic or Lie-Poisson integrator theory). We use the point of view of geometric quantization theory (orbit method) instead of harmonic analysis. Because of this we can consider all previous cases analogously.

Let $Sp(n)$ be symplectic group, $Mp(n)$ be its unique two- fold covering – metaplectic group. Let V be a symplectic vector space with symplectic form, then $R \oplus V$ is nilpotent Lie algebra - Heisenberg algebra, $Sp(V)$ is a group of automorphisms of Heisenberg algebra. Let N be a group with Lie algebra $R \oplus V$, i.e. Heisenberg group. By Stone– von Neumann theorem Heisenberg group have unique irreducible unitary projective representation. But this representation is unitary representation of universal covering, i.e. metaplectic group $Mp(V)$. We consider this representation without Stone-von Neumann theorem. Consider a new group $F = N' \bowtie Mp(V)$ ($\bowtie$ – semidirect product $N' = S^1 \times V$). Let V^* be dual to V, $G(V^*)$ be automorphism group of V^*. Then F is subgroup of $G(V^*)$, which consists of elements, which acts on V^* by affine transformations.

This is the key point!

Let $q_1, ..., q_n; p_1, ..., p_n$ be symplectic basis in V, $\alpha = pdq = \sum p_i dq_i$, $d\alpha$ be symplectic form on V^*, M is fixed affine polarization, then we can give the representation of infinitesimal basic elements. Lie algebra of the group F is the algebra of all (nonhomogeneous) quadratic polynomials of (p,q) relatively Poisson bracket (PB). The basis of this algebra consists of elements $1, q_1, ..., q_n, p_1, ..., p_n, \quad q_i q_j, q_i p_j, p_i p_j, \quad i, j = 1, ..., n, \quad i \leq j$, and we have the representation of basic elements, which gives the structure of the Poisson manifolds to representation of any (nilpotent) algebra or,in other words, to continuous wavelet transform. As particular cases we consider the Segal-Bargman representation, oscillator group, orbital theory, Kirillov character formula and point out reminiscence to wavelet analysis.

Then we can use obtained symplectic and Poisson structures in the formulae for symplectic Melnikov functions of Holmes-Marsden and Robinson–Koiller–Desolneaux-Moulis.

(m) **Oleg V. Vasilyev (Minnesota Supercomputer Institute)**

Adaptive multilevel wavelet collocation method for solving partial differential equations in multiple dimensions

Dynamically adaptive multilevel wavelet collocation method is developed for the solution of partial differential equations in multiple

dimensions. The method utilizes the classical idea of collocation with the multilevel wavelet approximation. The method is based on the use of a new fast wavelet collocation transform (FWCT) algorithm. Additional efficiency of the method istransform (FWCT) algorithm. Additional efficiency of the method is achieved by using cardinal wavelet bases. In this case the computational cost of the algorithm is independent of the dimensionality of the problem and is O(N), where N is the total number of collocation points. The method can handle general boundary conditions. The multilevel structure of the algorithm provides a simple way to adapt computational refinements to local demands of the solution. High resolution computations are performed only in regions where singularities or sharp transitions occur. Numerical results demonstrate the ability of the method to resolve localized structures such as shocks, which change their location and steepness in space and time. The algorithm is applied to a set of one- and two-dimensional test problems. Numerical results indicate that the algorithm is very competitive compared with classical methods as well as with adaptive wavelet Galerkin algorithms. In addition, it has distinctive advantages in the treatment of general boundary condition and nonlinearities when compared to the latter methods. Furthermore, the algorithm can utilize not only wavelets, but any suitable basis functions which have compact or essentially compact support in physical and spectral spaces.

(n) **Tim Flaherty (University of Pittsburgh)**
Multi-wavelets in Two Dimensions

Two constructions of 2-dimensional orthogonal multi-wavelets are given. Both constructions use existing 1-dimensional compactly supported continuous multi-wavelets. In the direct tensor product construction we get 4 continuous scaling functions, and 12 wavelets, defined on R2. In an indirect method I obtain 2 scaling functions and 6 wavelet functions on R2. The wavelets have small support, and generate an orthonormal basis of L2(R2). Furthermore, a calculation of the joint spectral radius of the appropriate transition matrices suggests that the wavelets are continuous.

(o) **Tian-Xiao He (Illinois Wesleyan University)**
Spline wavelet transform

In this paper, we will discuss window Fourier transforms, integral wavelets transforms, and wavelet series expansions associated with spline functions in shift-invariant spaces of B-splines. A recurrence relation formula and the corresponding algorithm about the B-wavelets will also be given.

(p) **Joe Ward (Texas A&M University)**
Nonstationary Wavelets on the m-Sphere for Scattered Data

We construct classes of nonstationary wavelets generated by what we call *sp herical basis functions* (SBF's), which comprise a subclass of Schoenberg's posi tive definite functions on the m-sphere. The wavelets are intrinsically defined on the m-sphere, and are independent of the choice of coordinate system. In addi tion, they may be orthogonalized easily, if desired. We will discuss decomposition, reconstruction, and localization for these wavelets. In the speci al case of the 2-sphere, we derive an uncertainty principle that expresses the t rade-off between localization and the presence of high harmonics–or high freque ncies– in expansions in spherical harmonics. We discuss the application of this principle to the wavelets that we construct.

(q) **Qing Gu (Texas A&M University)**
Existance of certain types of interpolation families of wavelet sets.

Certain types of interpolation families of wavelet sets are constructed.In particular,it's demonstrated that for any finite group there is an interpolation family of wavelet sets such that the set of all the measure preserving 2-homogeneous functions from R to R implementing the 2-pi translation congruency between any two wavelet sets in the interpolation family is a group isomorphic to that given finite group.Some questions concerning the V.N. Algebras related to the interpolation families of wavelet sets are also explored.

(r) **Wei Cai (UNC-Charlotte)**
Simulations of laminar flames with wavelet collocation methods.

There two major issues in the numerical simulations of flame propagation: one is the adaptive meshing needed to capture the rapidly changing solution, another is the ability of fast solution of the algebraic systems resulting from the implicit discretizations of the diffusion operators and chemical reaction terms in the reacting Navier-Stokes equations. We will report progresses in both areas for the study of adaptive wavelet schemes. Regarding adaptive meshing of wavelet methods, we have developed an efficient data structure for two dimensional wavelet approximations. We have utilized sparse matrix data structure (compressed row and compressed column vector technique) in treating the data structure on each wavelet space in the case of two dimensional approximations. Such an approach has the advantage of only storing the mesh points used in the adaptive wavelet approximations and easy access to numerical data on each constant x and y lines for the numerical differentiations.

On the issue of time integration, we have successfully implemented a second order implicit factorized scheme of Beam and Warming type for the adaptive wavelet methods. Unlike many other adaptive methods (finite element and finite difference), the adaptive wavelet methods actually can be easily implemented using an ADI approach. So, solution of the algebraic systems from the implicit discretization of 2-dimensional diffusion operators is replaced by that of only 1-dimensional operators, thus speeding up the time integrations tremendously.

(s) **Gustavo Garrigos (Washington University)**
Some connectivity results in wavelets.

We give some recent results concerning connectivity for the set of all wavelets in $L^2(R)$. In particular, we show how to find a path of wavelets that joins the two most classical wavelets: the Haar wavelet and the Shannon wavelet.

(t) **Ming-Jun Lai (University of Georgia-Athens)**
Examples of Bivariate Nonseparable Compactly Supported Orthonormal Wavelets

We present several examples of bivariate nonseparable compactly supported orthonormal wavelets. The Holder continuity properties of these wavelets are studied

(u) **Deguang Han (Texas A&M University)**
Irrational rotation unitary operator systems

We will give an abstract characterization for those irrational rotation unitary systems which have complete wandering vectors. A complete characterization of the wandering vector multiplier set for irrational rotation unitary systems is obtained.

(v) **Jeff Knisley (East Tennessee State University)**
A Method for Estimating the Amplitudes and Frequencies of Gabor Wavelet Decompositions

The windowed Fourier transform – in particular, the Fourier transform windowed with a guassian pulse – retains all of the physical interpretations of the general Fourier tran sform as was shown by Gabor in 1946. However, such windowed transforms have undesirable properties due to the uniformity of the window size and the necessity of "chopping up" the waveform so that orthogonality can

be used in the reconstruction. Conversely, Wavelets have the desirable prope rty of orthogonality but often have little or no physical interpretation.

We have found a reliable method for the decomposition of a signal into a wavelet expansion which does not rely on orthogonality. This method can be applied to most wavelet bases whi ch arise from coherent states. We will demonstrate its implementation and reliability for the windowed Fourier transform described by Gabor in 1946.

(w) **Zhiping He (Canada)**

Wavelet multipliers and Signals

The Schatten-von Neumann property of a pseudo-differential operator is established by showing that the pseudo-differential operator is a multiplier defined by means of an admissible wavelet associated to a unitary representation of the additive group $\mathbf{R}^n$ on the C^*-algebra of all bounded linear operators from $L^2(\mathbf{R}^n)$ into $L^2(\mathbf{R}^n)$. A bounded linear operator on $L^2(\mathbf{R})$ arising in the Landau, Pollak and Slepian model in signal analysis is shown to be a wavelet multiplier studied in this paper.

(x) **Carl Taswell (Stanford University)**

Wavelet Transform Convolution Versions for Fast Wavelet Based Numerical Algorithms

Beylkin et al (1) introduced fast O(N) and O(N log N) algorithms for the efficient multiplication of sparse wavelet based representations for integral and pseudodifferential matrix operators of a certain class. Keinert (20) continued this work by implementing the Beylkin algorithm for biorthogonal instead of orthogonal wavelets, and observing the relative advantages and disadvantages of the various wavelets investigated. In addition to the choice of wavelet filter, the other major component of a wavelet transform that must be specified for any implementation is the choice of a filter convolution version. Due to the finite size of the matrices, treatment of the boundaries and filter phase delays must be chosen. Typical alternatives are circularly-periodized, symmetrically-reflected, and boundary-adjusted convolution versions. New phase-aligned variants of these convolution versions are introduced and all of them are compared experimentally on the standard test matrices investigated in (1) and (2).

(1) Beylkin et al, 1991, Fast wavelet transforms and numerical algorithms, CPAM 44:141-183.

(2) Keinert, 1994, Biorthogonal wavelet for fast matrix computations, ACHA 1:147-156.

(y) **Jeff Hogan (Macquarie University, Australia)**
Wavelet Bi-frames with Unilateral Vanishing Moments

One method of constructing wavelet bi-frames is to write Riemann sum approximations of the Calderón-Zygmund singular integral operators which arise as inversion formulae (Calderón reproducing formulae) for the continuous wavelet transform. Ordinarily, the analysing and synthesising wavelets have at least one vanishing moment, and boundedness of the approximations (on $L^p(\mathbf{R}^n)$ $(1 < p < \infty)$, $H^1(\mathbf{R}^n)$ and $\mathrm{BMO}(\mathbf{R}^n)$) can be demonstrated through the use of classical techniques. We investigate the situation which arises when only one of the analysing allowlinebreak synthesising pair possesses a vanishing moment. It is shown that the dyadic discretisation of the continuous transform is in general not bounded. The $T(\mathbf{1})$-theorem is used to find delicate cancellation conditions on the analysing/synthsising pair which ensure boundedness and invertibility. The conditions are shown to depend on the particular regularisation of the singular integrals which perform the inversion. Almost everywhere convergence properties of the discretisations and the associated partial sum expansions are proved as consequences of the boundedness of the associated maximal functions.

(z) **Vishnu Kamat (Texas A&M University)**
Operators and Multi-wavelets

We study the set of wandering subspaces $\mathcal{WS}(\mathcal{U})$ for unitary systems, and in particular the unitary system $\mathcal{U}_{D,T}$ where D and T are 2-dilation and 1-translation unitary operators that generate $\mathcal{U}_{D,T}$ and act on $\mathcal{L}^2(\mathbf{R})$. The wandering subspaces for $\mathcal{U}_{D,T}$ are precisely the wandering subspaces generated by orthogonal multi-wavelets. We parameterize all wandering subspaces in $\mathcal{WS}(\mathcal{U})$ in terms of a fixed wandering subspace P in $\mathcal{WS}(\mathcal{U})$ and operators not necessarily unitary which locally commute with $\mathcal{U}$ at P. We further investigate the relationships between the wandering subspaces of different dimensions and the corresponding unitary system. We also generalize and study the concept of wavelet sets in 1-dimensions to joint wavelet sets in n dimensions that generate wandering subspaces for $\mathcal{U}_{D,T}$.